Stability analysis for the TPF interferometer

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4/2/03

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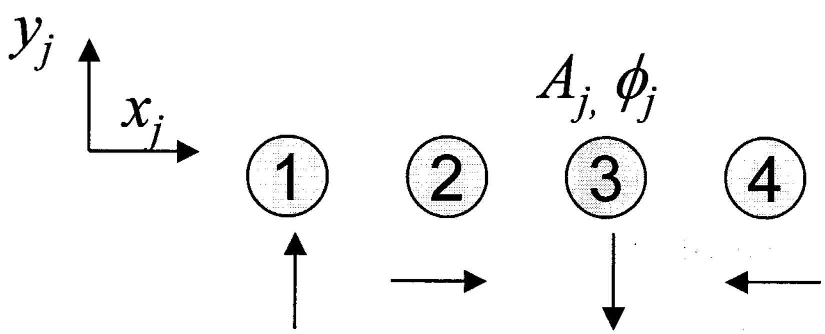
Summary

- New class of mixed "bi-linear" errors identified which dominate the stability budget
- Not removed by phase chopping
- Leads to tolerances ~ 5 times tighter than those needed for 10^{-5} null depth:
  - Amplitude control ~ 0.1%
  - Phase control ~ 1 nm
  - Approx. equivalent to requirements for 5×10^{-7} null depth
- Non-linear frequency mixing makes these difficult to calibrate
- Dual Bracewell used as example, but basic results apply to other configurations

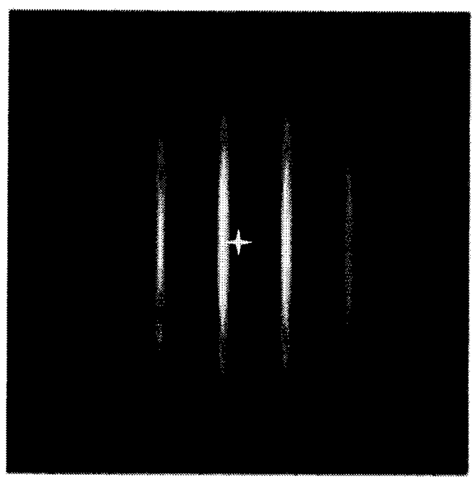
Scope

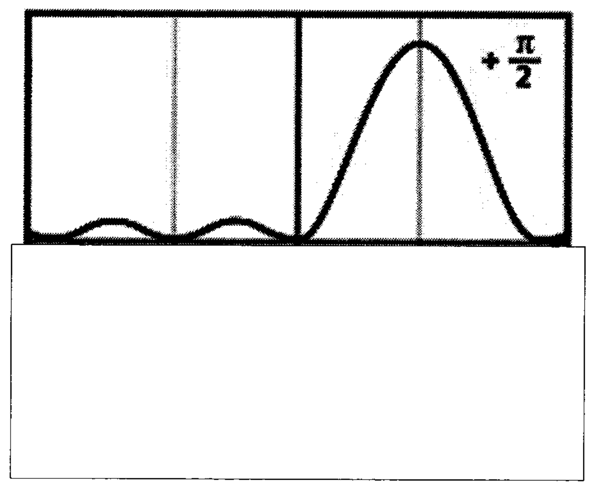
- Ability to isolate planet photons depends on:
  - Photon Shot noise
  - Detector gain variations
  - Thermal noise and scattered light
  - Polarization leakage
  - Null instability from E-field amplitude and phase imbalance
- This presentation is about the amplitude and phase balance. Contributors include:
  - Mirror surface figure
  - Pointing control
  - Delay tracking
  - Contamination of reflectivity
  - Dispersion effects
- The goal of this talk is to describe the new challenging requirements that have emerged

Dual Bracewell example



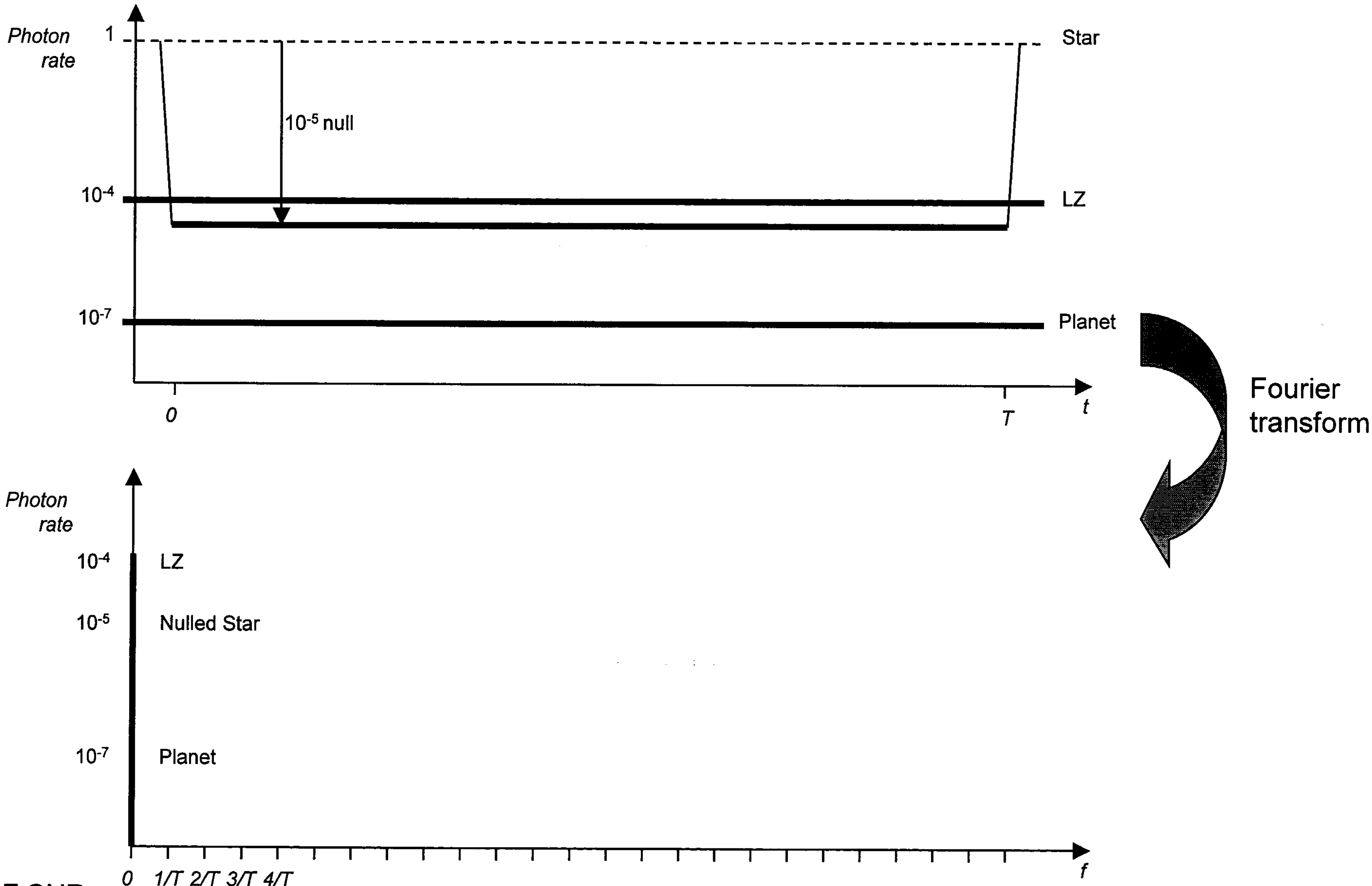
j	1	2	3	4
A_j	1	1	1	1
ϕ_j	0	$\pi/2$	π	$3\pi/2$



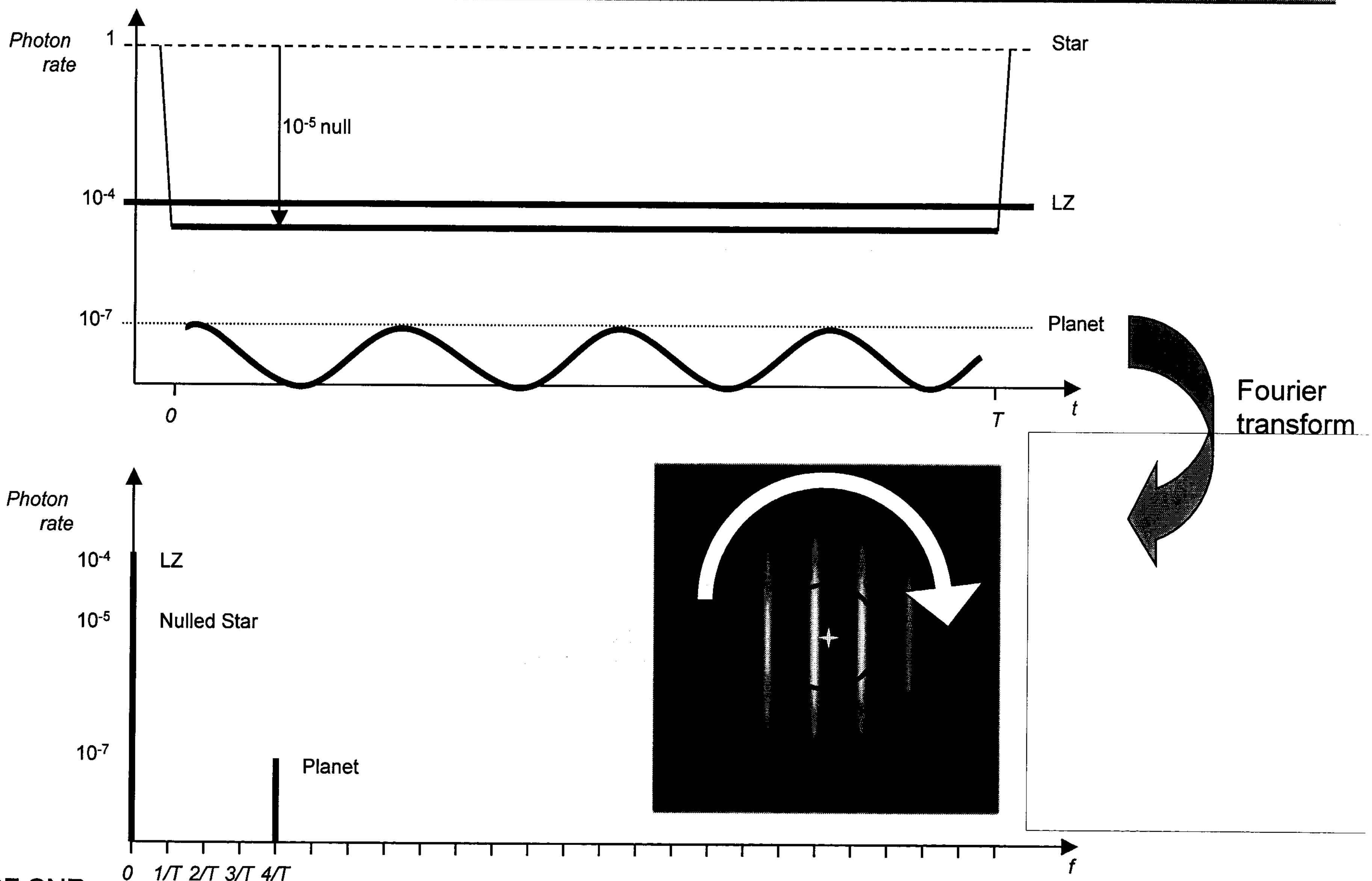


Prior view of stability requirements

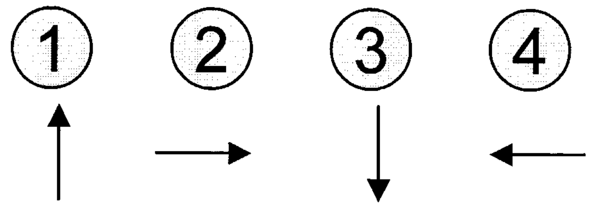
Photon rates

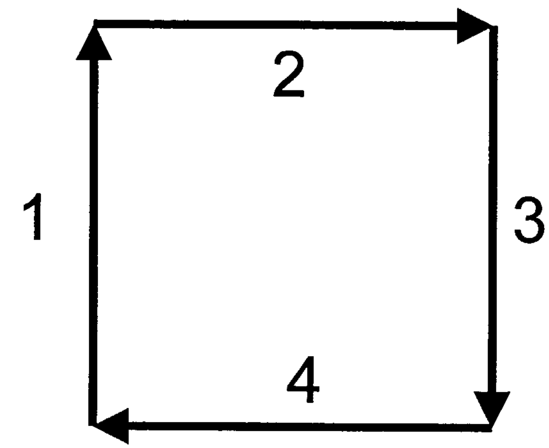


Rotate array to modulate planet signal

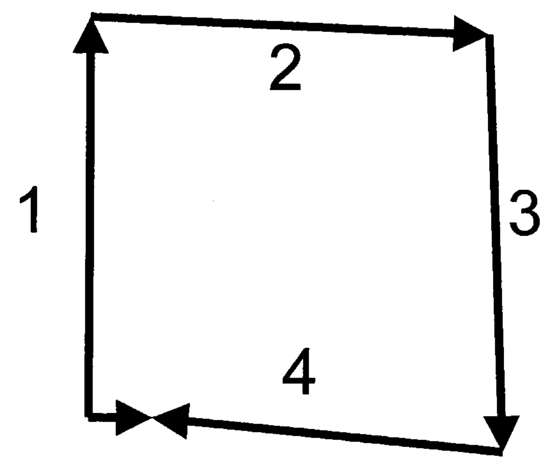


Null instability



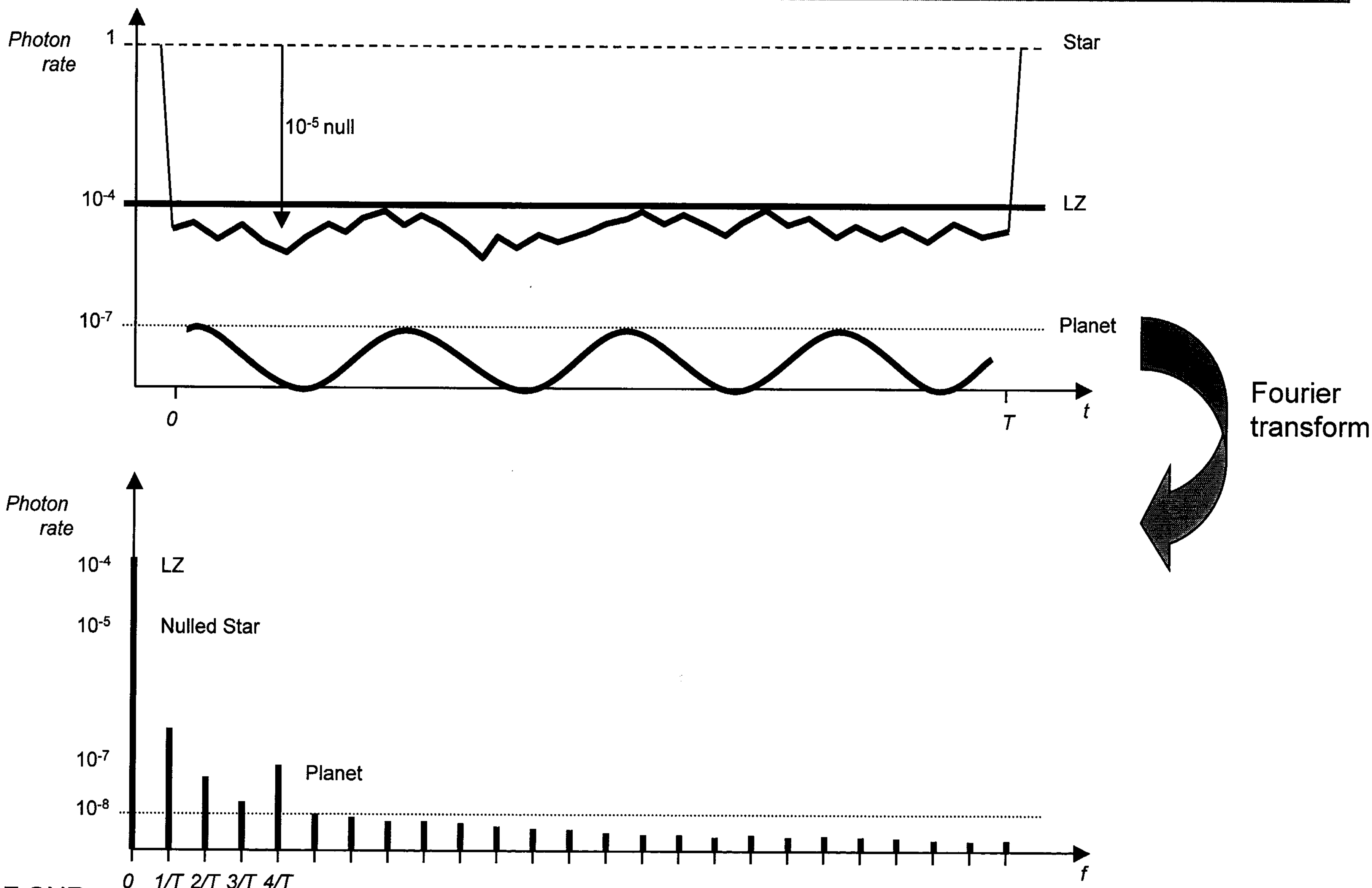


- A precarious balance of amplitudes and phases is holding back a deluge of stellar photons

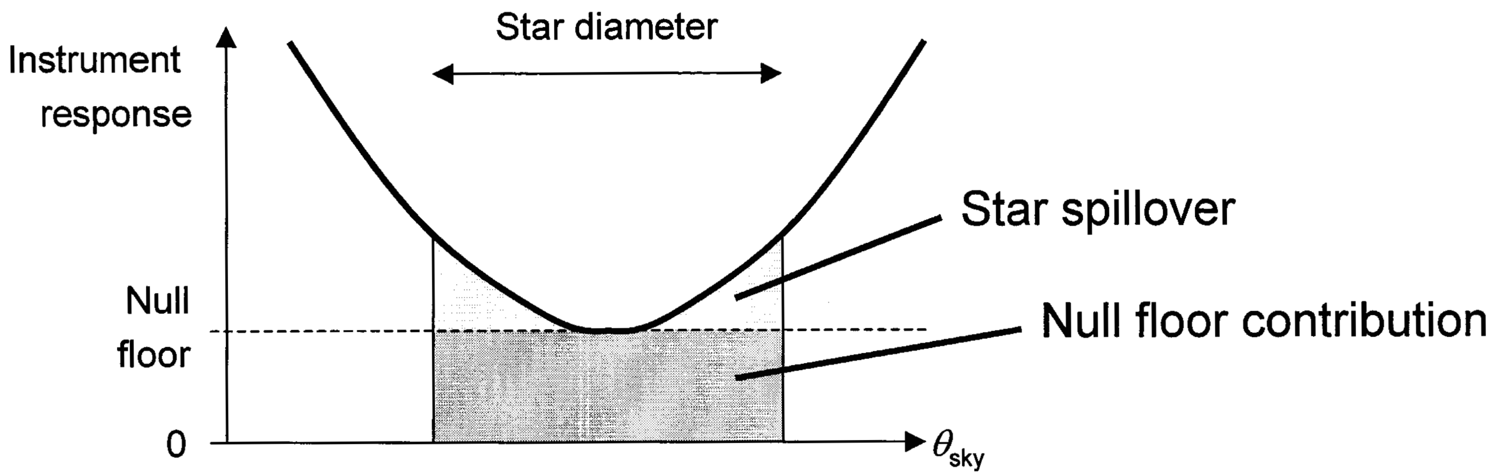


- Any small perturbation in amplitude or phase changes number of stellar photons
  - e.g. mis-pointing, vibration, alignment drift, distortion of optical surfaces

With amplitude & phase instability

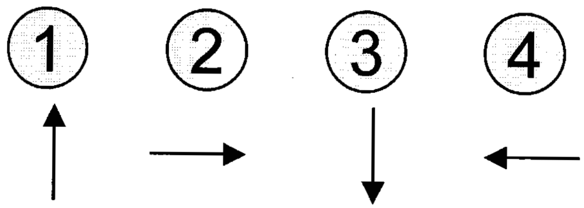


Stellar Leakage

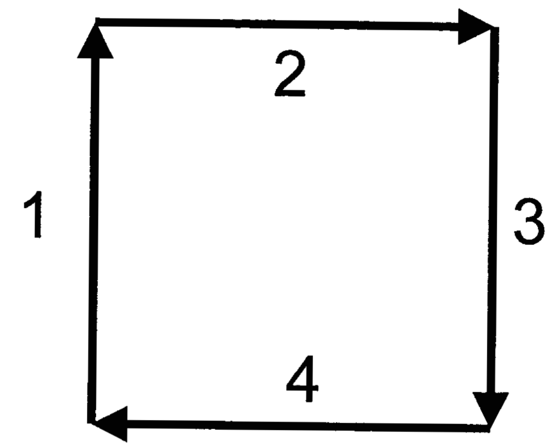


- Analysis covers both types of leakage
- "Null floor" effects dominate "Star spillover"
  - Tolerances derived here will be same for a point-like star
- A broad null does not help
  - Bracewell, OASES, etc. all approx. equally susceptible to amplitude & phase instabilities

Null instability



- A precarious balance of amplitudes and phases is holding back a deluge of stellar photons



- Any small perturbation in amplitude or phase changes number of stellar photons
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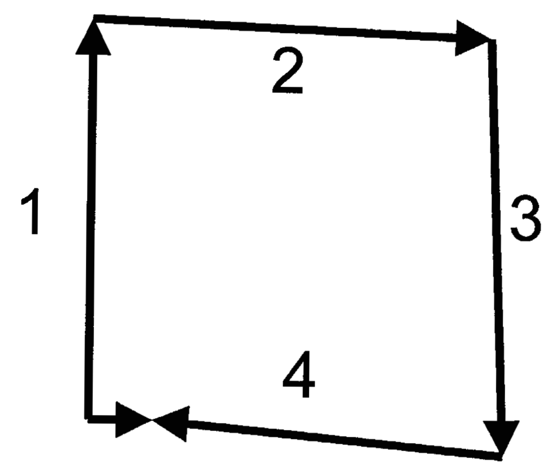
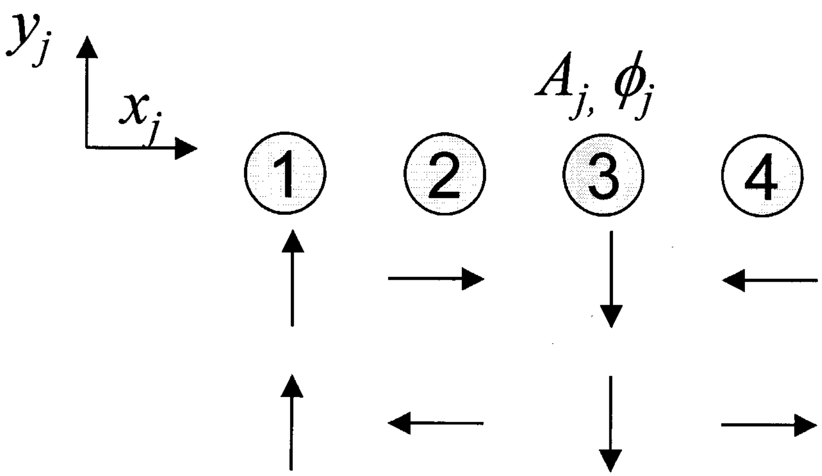
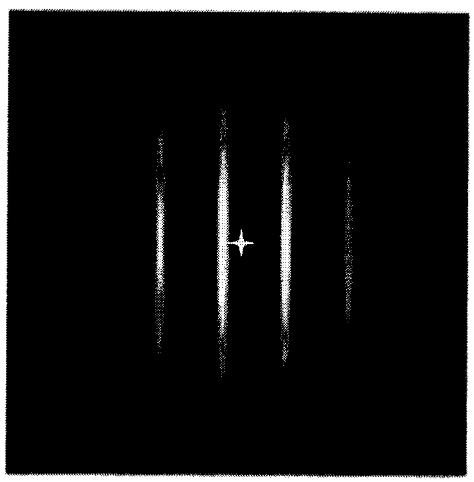
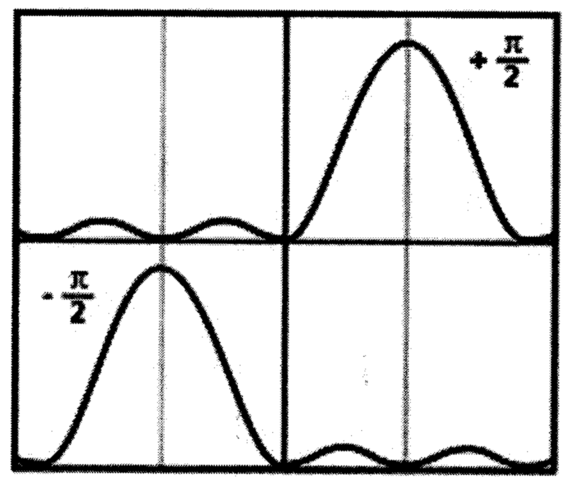


Dual Bracewell with chopping

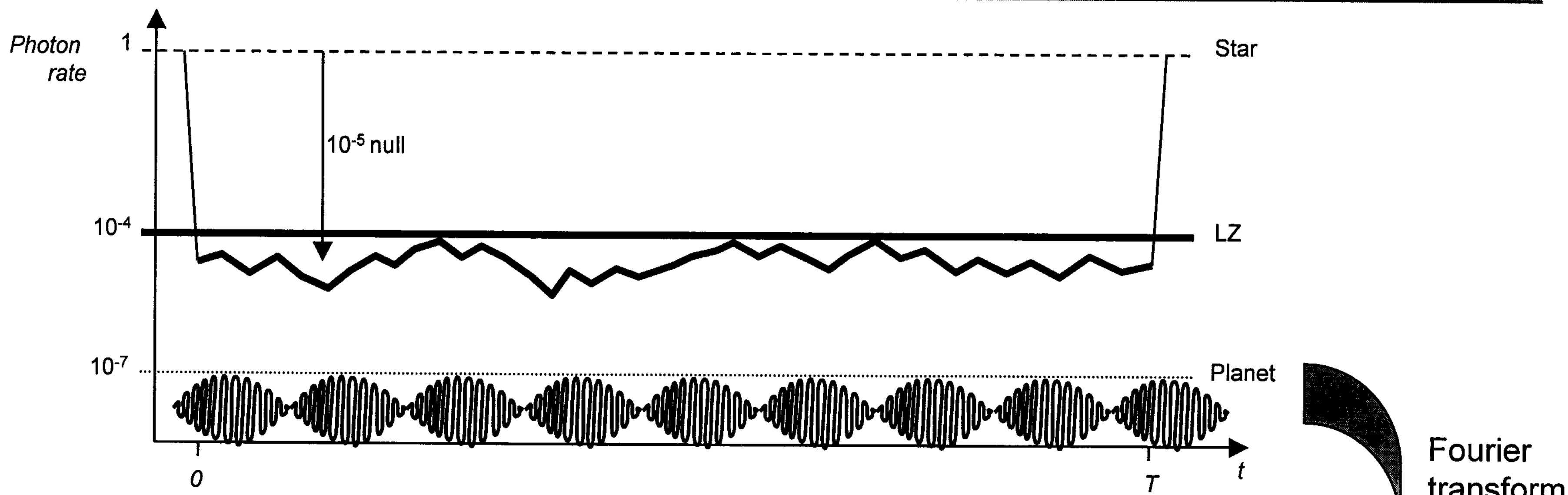


j	1	2	3	4
A_j	1	1	1	1
ϕ_j	0	$\pi/2$	π	$3\pi/2$
ϕ_j'	0	$3\pi/2$	π	$\pi/2$

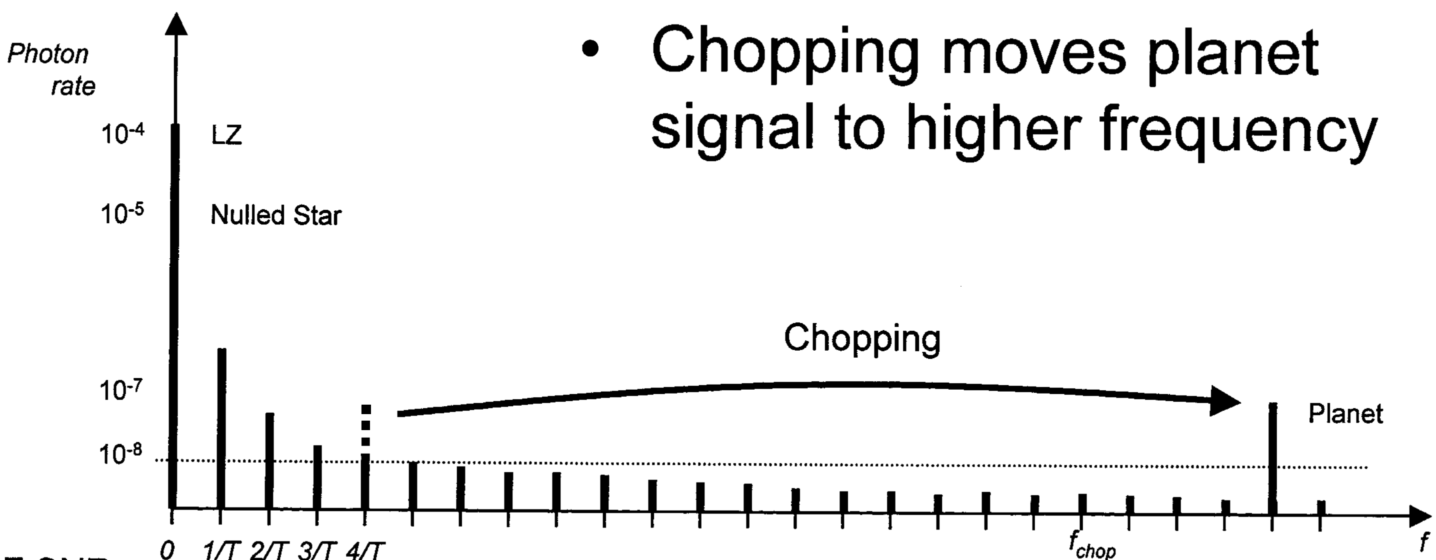




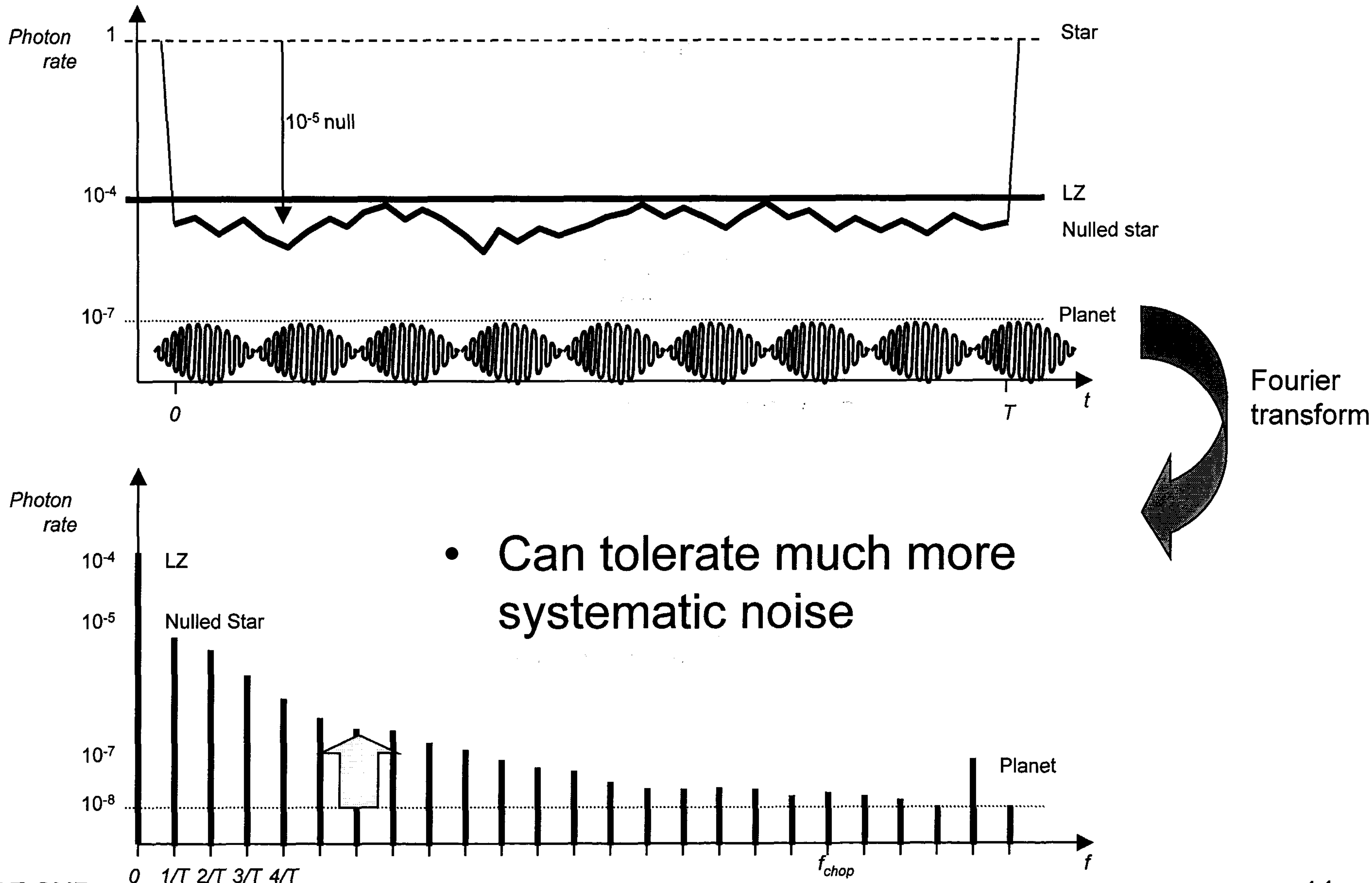
With chopping



- Chopping moves planet signal to higher frequency



With chopping



- Can tolerate much more systematic noise

Chopping requirements

		Old	
		No chopping	With chopping
Photon noise (null depth)	δA	0.5%	0.5%
	$\delta\phi$	7 nm	7 nm
Systematic noise (null stability)	δA	0.13%	4%
	$\delta\phi$	2.0 nm	60 nm

Assumes 1/f noise

- Photon noise drives requirements on amplitude and phase error
- Assumes chop action does not introduce asymmetry

New view of stability requirements

δX: first order terms

- Stellar photon rate

$$X\left(\left\{A_j, \phi_j, x_j, y_j\right\}\right)$$

- Sensitivity of photon rate to perturbations of the variables obtained by differentiation:

$$\delta X \approx \sum_j \left\{ \frac{dX}{dA_j}\delta A_j + \frac{dX}{d\phi_j}\delta\phi_j + \frac{dX}{dx_j}\delta x_j + \frac{dX}{dy_j}\delta y_j \right\}$$

- But $\{A_j, \phi_j\}$ have been chosen to minimize *X*, so these first derivatives are close to zero:

$$\frac{dX}{dA_j} \approx 0 \qquad \frac{dX}{d\phi_j} \approx 0$$

- Need to go to second order in these terms…

δX: second order terms

$$\cancel{\delta X \approx \sum_j \left\{ \frac{dX}{dA_j}\delta A_j + \frac{dX}{d\phi_j}\delta\phi_j + \frac{dX}{dx_j}\delta x_j + \frac{dX}{dy_j}\delta y_j + \frac{d^2X}{dA_j^2}\delta A_j^2 + \frac{d^2X}{d\phi_j^2}\delta\phi_j^2 \right\}}$$

- Need to include the mixed 'bi-linear' terms, of which there are many:

$$\delta X \approx \sum_j \left\{ \begin{array}{l} \frac{dX}{dA_j}\delta A_j + \frac{dX}{d\phi_j}\delta\phi_j + \frac{dX}{dx_j}\delta x_j + \frac{dX}{dy_j}\delta y_j + \\ \sum_k \left[\frac{1}{2}\frac{d^2X}{dA_j dA_k}\delta A_j \delta A_k + \frac{d^2X}{dA_j d\phi_k}\delta A_j \delta\phi_k + \frac{1}{2}\frac{d^2X}{d\phi_j d\phi_k}\delta\phi_j \delta\phi_k \right] \end{array} \right\}$$

- For 4 collectors we have 64 terms instead of 16

Breakdown by error type

- Breakdown of noise contributions (% of total noise variance):

$\{\delta A_j\}$	5%
$\{\delta \phi_j\}$	1%
$\{\delta A_j \delta A_k\}$	20%
$\{\delta A_j \delta \phi_k\}$	53%
$\{\delta \phi_j \delta \phi_k\}$	20%
$\{\delta x_j\}$	1%
$\{\delta y_j\}$	0%

- Lo-res Dual Bracewell configuration, 40 m array length
- Solar system @ 10 pc
- Single spectral channel @ 10 μm
- Full rotation of the array
- δA = 0.0005 (0.05%)
- $\delta\phi$ = 0.0005 rad (0.8 nm)
- δx = 0.01 m
- δy = 0.01 m
- Gives SNR = 2 (systematic noise only)

- Dominated by mixed, bi-linear terms, particularly amplitude-phase

Chopping revisited

- Phase chopping removes some errors
- But is ineffective against the new, dominant, mixed amp-phase terms

- Breakdown of noise contributions (% of total noise variance):

$\cancel{\{\delta A_j\}}$	~~5%~~
$\{\delta\phi_j\}$	1%
$\cancel{\{\delta A_j \delta A_k\}}$	~~20%~~
$\{\delta A_j \delta\phi_k\}$	53%
$\cancel{\{\delta\phi_j \delta\phi_k\}}$	~~20%~~
$\cancel{\{\delta x_j\}}$	~~1%~~
$\{\delta y_j\}$	0%

Graphical examples

Error term	Chop state	Collector				Resultant phasor	Photon rate	Left - Right
		1	2	3	4			
δA_1^2	Left	$\uparrow$ ε	$\downarrow$	$\rightarrow$	$\leftarrow$	$\uparrow$ ε	ε^2	0
	Right	$\uparrow$ ε	$\downarrow$	$\leftarrow$	$\rightarrow$	$\uparrow$ ε	ε^2	

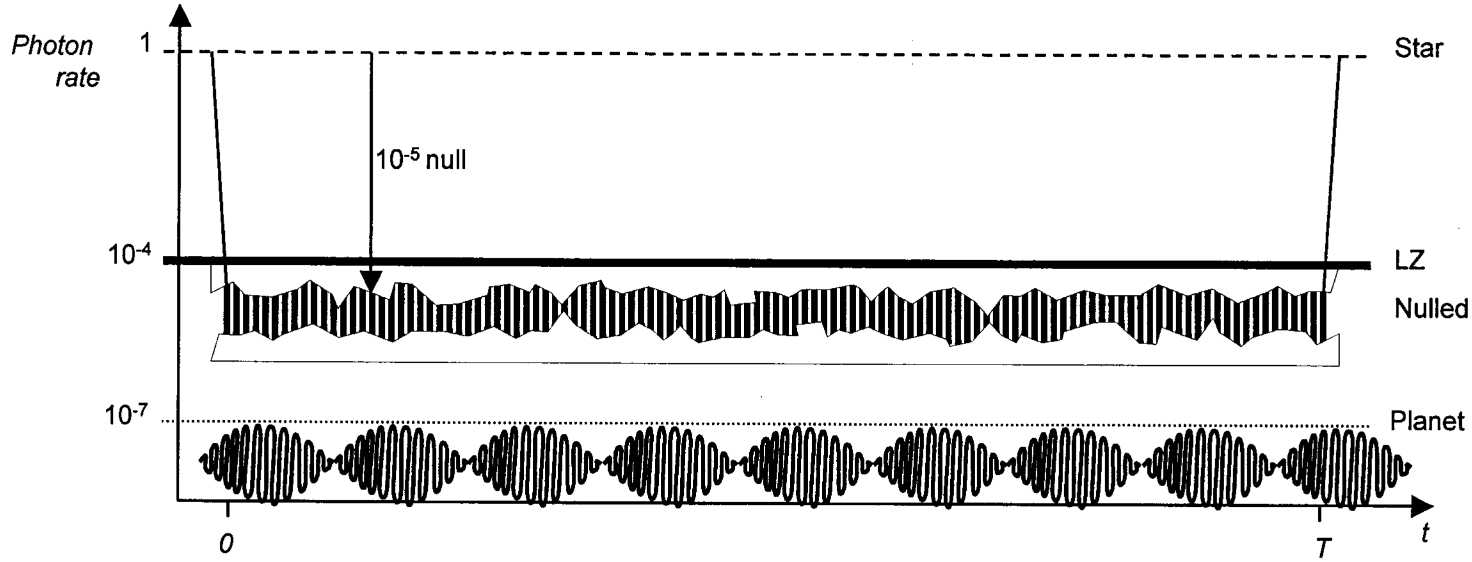
- This quadratic error is effectively suppressed by phase chopping

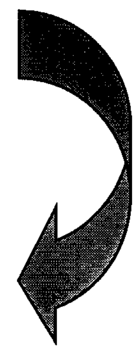
Graphical examples

Error term	Chop state	Collector				Resultant phasor	Photon rate	Left - Right
		1	2	3	4			
δA_1^2	Left	$\uparrow$ ε	$\downarrow$	$\rightarrow$	$\leftarrow$	$\uparrow$ ε	ε^2	0
	Right	$\uparrow$ ε	$\downarrow$	$\leftarrow$	$\rightarrow$	$\uparrow$ ε	ε^2	
$\delta A_1 \delta\phi_3$	Left	$\uparrow$ ε	$\downarrow$	$\rightarrow$ $\uparrow$ ε	$\leftarrow$	2ε $\uparrow$	$4\varepsilon^2$	$4\varepsilon^2$
	Right	$\uparrow$ ε	$\downarrow$	$\leftarrow$ $\downarrow$ ε	$\rightarrow$	•	0	

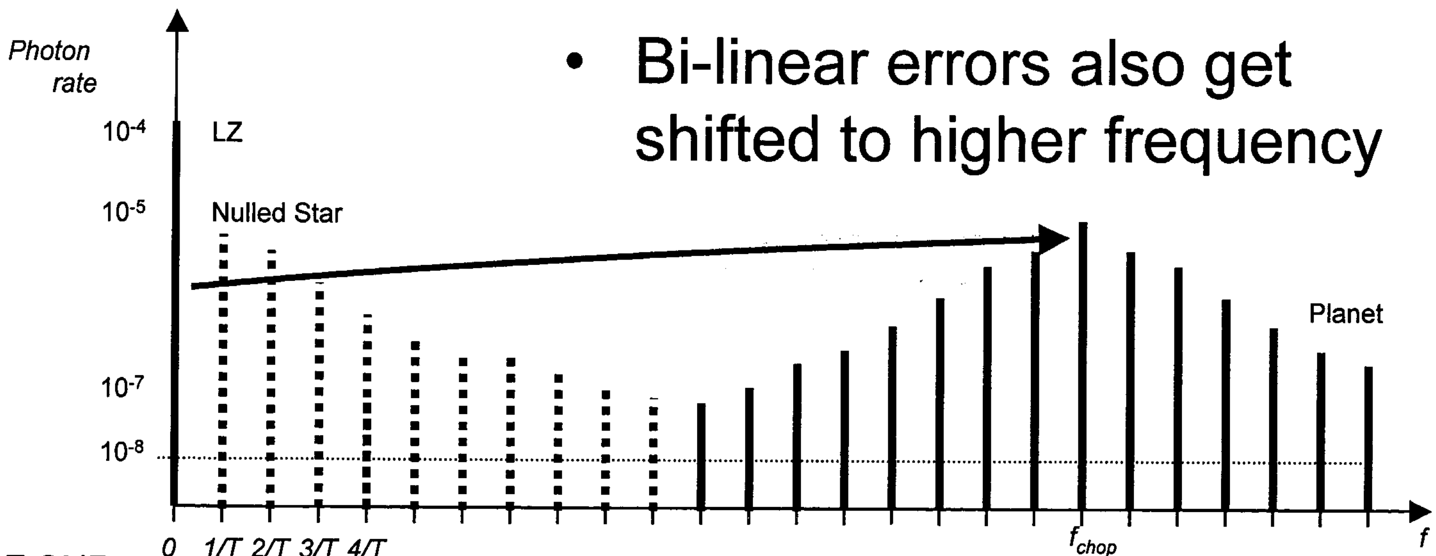
- …but this bi-linear error is amplified

With chopping

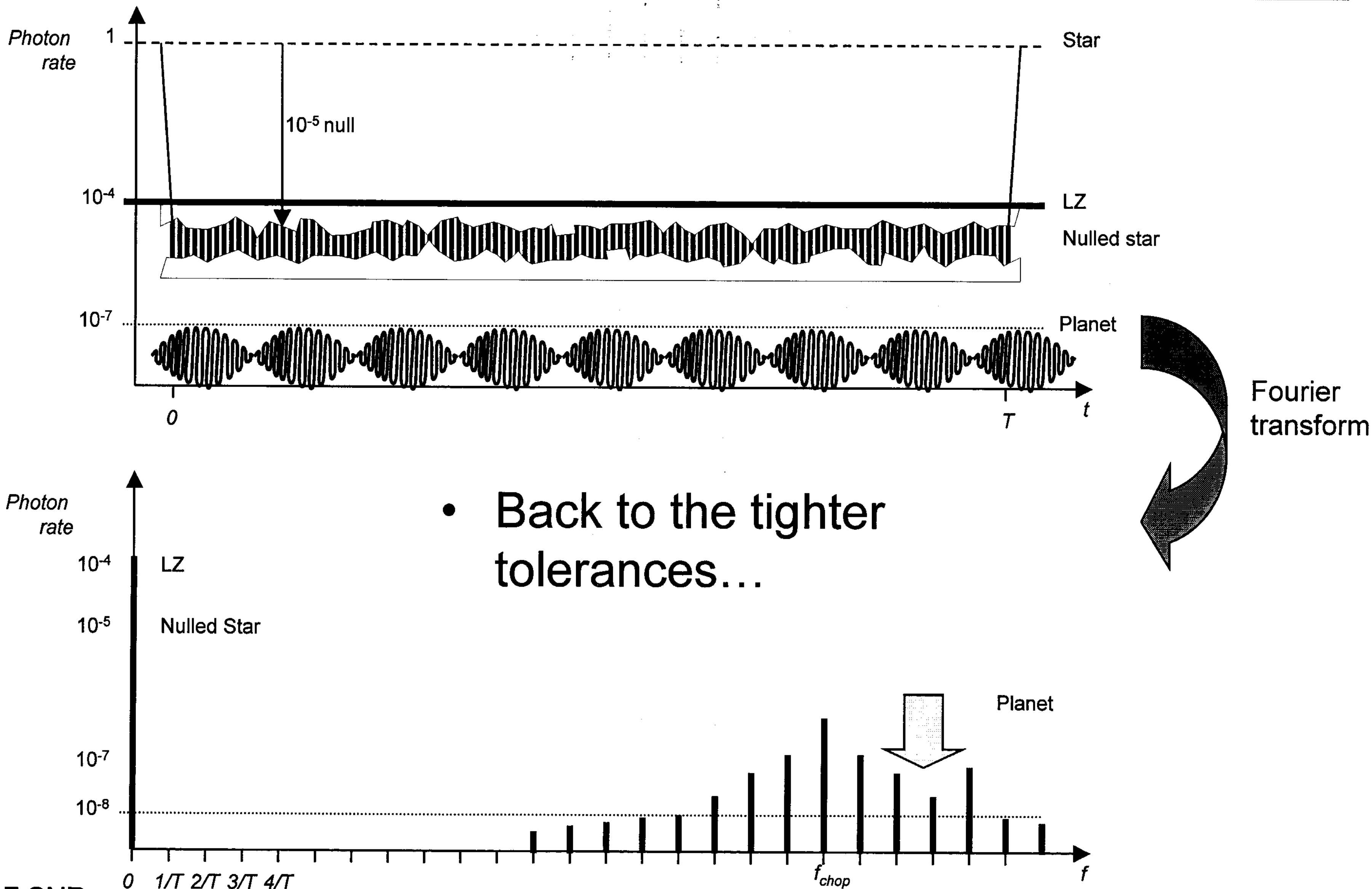




- Bi-linear errors also get shifted to higher frequency



With chopping



- Back to the tighter tolerances…

Change in requirements

		Old		New	
		No chop	Chop	No chop	Chop
Photon noise (null depth)	δA	0.5%	0.5%	0.5%	0.5%
	$\delta\phi$	7 nm	7 nm	7 nm	7 nm
Systematic noise (null stability)	δA	0.13%	4%	0.09%	0.1%
	$\delta\phi$	2.0 nm	60 nm	1.4 nm	1.5 nm

- Systematic noise drives requirements on amplitude and phase error, with or without phase chopping

Based on detecting Earth @ 10 pc, Dual Chopping Bracewell

Non-linear complications

$$\delta X \approx \sum_j \left\{ \begin{array}{l} \frac{dX}{dA_j}\delta A_j + \frac{dX}{d\phi_j}\delta\phi_j + \frac{dX}{dx_j}\delta x_j + \frac{dX}{dy_j}\delta y_j + \\ \sum_k \left[\frac{1}{2}\frac{d^2X}{dA_j dA_k}\delta A_j \delta A_k + \frac{d^2X}{dA_j d\phi_k}\delta A_j \delta\phi_k + \frac{1}{2}\frac{d^2X}{d\phi_j d\phi_k}\delta\phi_j \delta\phi_k \right] \end{array} \right\}$$

- Null stability is dominated by non-linear terms
- For a linear term:
  - fluctuations in A_1 at 0.1 mHz cause fluctuations in photon rate at 0.1 mHz
- Bi-linear terms: mixing between perturbations!!
  - fluctuation in A_1 at 5.4 mHz and a fluctuation in ϕ_3 at 5.3 mHz mix to give a fluctuation in X at 0.1 mHz
- Entire PSD for amplitude and phase contributes to each fluctuation frequency in photon rate
- Means that regular calibration of amplitude and phase has limited effect

Why are these requirements hard?

- They are requirements on control, not just knowledge
- They apply to all frequencies, including DC
  - not a particular frequency range
  - PSD shape has some impact
- They apply across a factor of 3 in wavelength and to both polarizations
- Tolerances relax only as $T^{1/4}$
- Cannot use null depth for sensing
  - Not enough SNR to detect variations 5 times weaker than planet
- So must measure amplitudes for individual beams and phases between pairs

		New	
		No chop	Chop
Systematic noise (null stability)	δA	0.09%	0.1%
	$\delta\phi$	1.4 nm	1.5 nm

What now?

3 possibilities:

- Show that new analysis is incorrect
- Find an observable for a nulling configuration that is much less sensitive to amplitude and phase perturbations
- Identify an approach to controlling amplitude to 0.1% and phase to ~1.5 nm

Summary

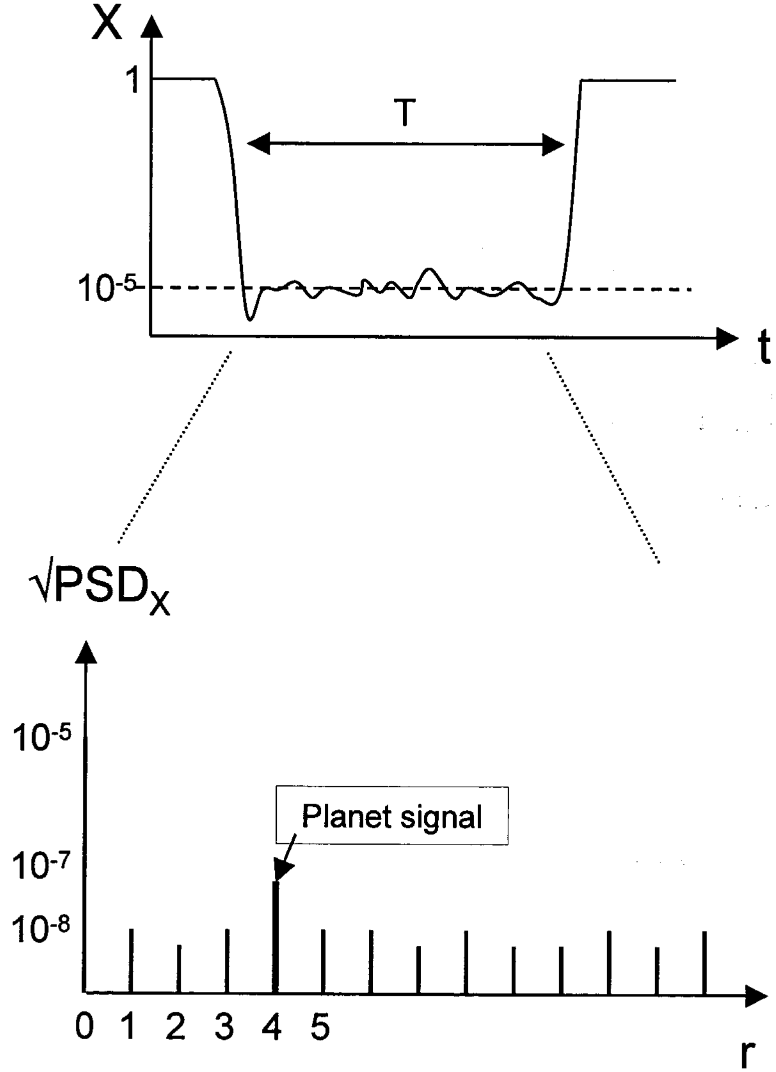
- New class of mixed "bi-linear" errors identified which dominate the stability budget
- Not removed by phase chopping
- Leads to tolerances ~ 5 times tighter than those needed for 10^{-5} null depth:
  - Amplitude control ~ 0.1%
  - Phase control ~ 1 nm
  - Approx. equivalent to requirements for 5×10^{-7} null depth
- Non-linear frequency mixing makes these difficult to calibrate
- Dual Bracewell used as example, but basic results apply to other configurations

Back-up slides

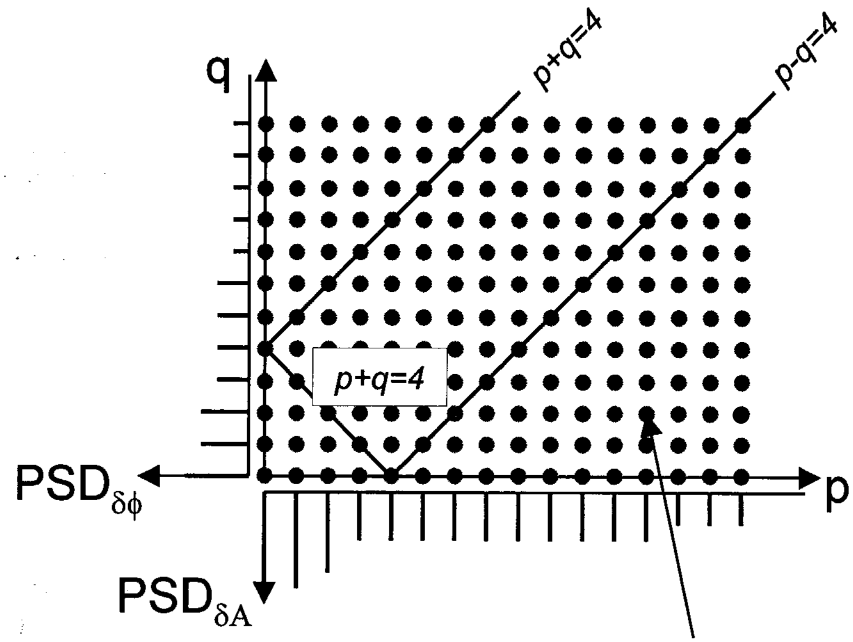
Mitigation options considered

- Rapid rotation of the array
  - Tolerances on δA and $\delta\phi$ go as $f_{rot}^{-1/4}$
- Regular monitoring and correction of A and ϕ
  - Perfect calibration every 100 s only relaxes tolerances by factor ~2.5
- Use full 0-2π sweep of phase before cross-combiner
  - Does not help since planet and star both have same sinusoidal signature

Power Spectra



$\delta A \delta \phi$ contribution:



$\propto PSD_{\delta A}(p)\,PSD_{\delta\phi}(q)$

$$\delta X \approx \sum_j \left\{ \sum_k \left[\frac{1}{2} \frac{d^2 X}{dA_j dA_k} \delta A_j \delta A_k + \frac{d^2 X}{dA_j d\phi_k} \delta A_j \delta\phi_k + \frac{1}{2} \frac{d^2 X}{d\phi_j d\phi_k} \delta\phi_j \delta\phi_k \right] \right\}$$

$$X\left(\left\{A_j, \phi_j, x_j, y_j\right\}\right) \qquad \delta X \approx \sum_j \left\{ \frac{dX}{dA_j} \delta A_j + \frac{dX}{d\phi_j} \delta\phi_j + \frac{dX}{dx_j} \delta x_j + \frac{dX}{dy_j} \delta y_j \right\}$$

$$\frac{dX}{dA_j} \approx 0 \qquad \frac{dX}{d\phi_j} \approx 0$$

$$\delta X \approx \sum_j \left\{ \frac{dX}{dA_j} \delta A_j + \frac{dX}{d\phi_j} \delta\phi_j + \frac{dX}{dx_j} \delta x_j + \frac{dX}{dy_j} \delta y_j + \frac{d^2 X}{dA_j^{\,2}} \delta A_j^{\,2} + \frac{d^2 X}{d\phi_j^{\,2}} \delta\phi_j^{\,2} \right\}$$

$$\delta X \approx \sum_j \left\{ \begin{array}{l} \dfrac{dX}{dA_j} \delta A_j + \dfrac{dX}{d\phi_j} \delta\phi_j + \dfrac{dX}{dx_j} \delta x_j + \dfrac{dX}{dy_j} \delta y_j + \\ \displaystyle\sum_k \left[\frac{1}{2} \frac{d^2 X}{dA_j dA_k} \delta A_j \delta A_k + \frac{d^2 X}{dA_j d\phi_k} \delta A_j \delta\phi_k + \frac{1}{2} \frac{d^2 X}{d\phi_j d\phi_k} \delta\phi_j \delta\phi_k \right] \end{array} \right\}$$